ON THE OPTIMUM WIDTH OF THE SEPARATING GAP IN LIQUID THERMODIFFUSION COLUMNS

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Based on earlier developed models of the partition process with attendant stray convection, the authors make recommendations on how the efficiency of liquid thermodiffusion columns may be raised.

For steady-state operation, as is well known, the logarithm of the partition index in a Clausius–Dickel apparatus is

$$\ln q^* = \frac{HL}{K_s + K_d}, \tag{1}$$

with L denoting the column length and the coefficients H, K_s, K_d in the transport equation [1] for liquid isotope mixtures are

$$H = \frac{\alpha g \beta \rho^2 \delta^3 (\Delta T)^2 B}{6!\, \eta T}, \tag{2}$$

$$K_s = \frac{g^2 \beta^2 \rho^3 \delta^7 (\Delta T)^2 B}{9!\, \eta^2 D}, \tag{3}$$

$$K_d = B \delta \rho D. \tag{4}$$

It follows from an analysis of (1)-(4) that the dimension of the separating gap δ has an appreciable effect on the attainable magnitude of the partition index: as δ is decreased, this index first increases and then decreases. The maximum value of $\ln q^*$ is reached when

$$\delta_{opt} = 7.5 \left[\frac{\eta D}{g \beta \rho (\Delta T)} \right]^{\frac{1}{3}} \tag{5}$$

and is determined from the expression

$$(\ln q^*)_{max} = 0.393\, \frac{\alpha (\Delta T)}{T} \cdot \frac{L}{\delta_{opt}}. \tag{6}$$

Inserting for the parameters in (5) their orders of magnitude characteristic of liquids $\eta \approx 10^{-3}$ N·sec/m^2, D $\approx 10^{-9}$ m^2/sec, $\beta \approx 10^{-3}$ deg^{-1}, $\rho \approx 10^3$ kg/m^3, and $\Delta T \approx 10^2$ deg will yield $\delta_{opt} \approx 0.075$ mm, which, according to (6), will correspond to a very high partition index $q^* = 400$ in an only 0.4 m long column with $\alpha = 0.01$.

At the same time, it has never been possible to attain such a high partition index in a liquid thermodiffusion apparatus under such conditions because it is not technically feasible to make both surfaces isothermal in a column for which expression (1) would be valid.

The anisothermy results in a so-called stray convection and while the authors of [1], who first noted these detrimental effects, had only considered the departure from isothermism due to deviations of the column geometry from an ideal one, we will here consider also the important role played by the mode of heat supply and heat removal.

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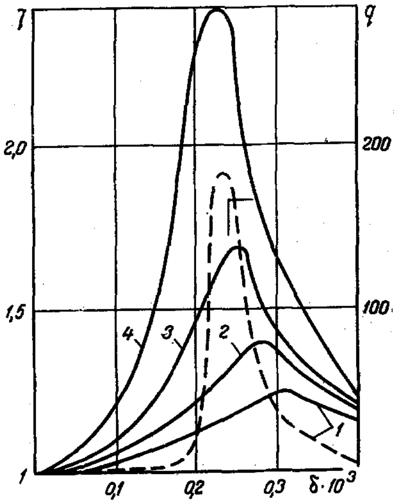

Fig. 1. Partition index q in a thermodiffusion column, as a function of the separating gap width δ under various conditions of convective heat transfer: 1) $\alpha L = 10^{-2}$ m (solid lines), $\alpha L = 10^{-1}$ m (dashed line), $s = 2.5 \cdot 10^4$ m^{-1}; 2) $5 \cdot 10^4$ m^{-1}; 3) 10^5 m^{-1}; 4) $2 \cdot 10^5$ m^{-1}.

A theory has been presented in [2] which quantitatively describes the effect of convection on the partition process and on the basis of which, as will be shown here, one can determine the optimal dimensions of the separating gap so as to indicate ways of improving the efficiency of liquid thermodiffusion columns.

For the case where one of the components appears in low concentrations $c \ll 1$, the partition index with stray convection in the column is defined by the expression

$$q = \frac{1+\varkappa}{1-\varkappa} \cdot \frac{1-2\varkappa+\exp\left[(1-\varkappa)\,y_e\right]}{1+2\varkappa+\exp\left[-(1+\varkappa)\,y_e\right]}, \tag{7}$$

where

$$\varkappa = 15\,\frac{T\,(\delta T)}{\alpha\,(\Delta T)^2} \tag{8}$$

is a parameter characterizing the effect of stray convection and δT is the azimuthal temperature nonuniformity.

When heat is supplied from an electric heater, it can be shown that

$$(\delta T)_{el} = \frac{\varepsilon}{\delta}\,(\Delta T), \tag{9}$$

where ε is the mean deviation from the nominal gap size. When heat is supplied through convection, however, then

$$(\delta T)_{conv} = \frac{\varepsilon}{\delta} \cdot \frac{(\Delta T)}{1+\delta h_2/\left[\lambda\,(1+h_2/h_1)\right]}, \tag{10}$$

where $h_1 = \alpha_1/(1+Bi_1)$, $h_2 = \alpha_2/(1+Bi_2)$, Bi_1 and Bi_2 are values of the Biot number at the hot wall and at the cold wall of the apparatus respectively, α_1 and α_2 are the heat transfer coefficients at both thermostatic column surfaces, and λ is the thermal conductivity of the liquid filling the gap.

It is evident, according to (9) and (10), that electric heating always results in a greater temperature asymmetry and, therefore, is a less desirable mode of heating. An examination of formula (10) reveals also that a higher heat-transfer coefficient and a higher thermal conductivity of the walls yield less temperature asymmetry and may even reduce it to perfect symmetry in the extreme, thus creating the conditions under which formulas (1) and (6) are valid. An attempt to achieve such low values of the Biot number may prove to be undesirable, from the energy as well as the design standpoint, so that the practical operation of a thermodiffusion column is characterized by moderate values of the Biot number.

Moreover, as has been shown experimentally by the authors of [3], the partition index becomes maximum when the gap is wider than according to expression (5). With the aid of the formulas derived here, this experimental correction factor can be interpreted quantitatively.

For this we rewrite (8) taking into consideration (10):

$$\varkappa = \frac{r}{\delta\,(1+s\delta)},$$

where

$$r = 15\,\frac{\varepsilon T}{\alpha\,(\Delta T)}, \qquad s = \frac{h_1 h_2}{\lambda\,(h_1+h_2)}.$$

For $y_e = \ln q^*$, with (1)-(4) taken into account, we have

$$y_e = \frac{a\delta^2}{b+\delta^6}\,\alpha L,$$

where

$$a = 504 \frac{\eta D}{g\beta\rho T},$$

$$b = 9! \left[\frac{\eta D}{g\beta\rho (\Delta T)} \right]^2.$$

In subsequent calculations we assume the following values for parameters r, a, and b which are typical of thermodiffusion processes in liquid isotope mixtures: $r = 1.2 \cdot 10^{-2}$ m, $a = 5 \cdot 10^{-13}$ m^3, and $b = 3.6 \cdot 10^{-25}$ m^6. Then, as is evident from (7), the partition index obtained with stray convection taken into account is a function of three parameters: s, αL, and δ.

This relation is shown graphically in Fig. 1 and, according to it, within a wide range of variation in the mode of convective heat transfer, the optimum operating condition in a liquid thermodiffusion apparatus corresponds to a separating gap $\delta = 0.23$-0.30 mm wide. As the column length is increased at a constant thermal diffusivity, the maximum partition index shifts toward wider gaps while, conversely, increasing the thermal diffusivity at a constant column length will shift this maximum toward narrower gaps, as indicated by the dashed curve corresponding to a thermal diffusivity 10 times higher than that to which the solid curve 1 corresponds.

The maximum of this curve, as seen in Fig. 1, corresponds to $\delta = 0.24$ mm. The dashed curve may be considered characteristic for processes in binary liquid mixtures with a Soret coefficient of the order of $5 \cdot 10^{-3}$ deg^{-1} at rather moderate heat transfer coefficients.

It is to be noted, in conclusion, that the results shown here apply to low concentrations $c \ll 1$ but may be extended to higher concentrations too – based on the earlier analysis of stray convection in a thermodiffusion column [2].

NOTATION

q, q^*	are the partition index during steady state without and with stray convection in the column;
α	is the thermal diffusivity;
α_1, α_2	are the heat transfer coefficients at the thermostatic column surfaces;
$Bi_i = \alpha_i \delta_i / \lambda_i$ $(i = 1, 2)$	is the Biot number;
δ_i	is the thicknesses of the walls;
λ_i	is the thermal conductivities of the walls;
δ	is the width of the separating gap;
λ	is the thermal conductivity of liquid filling the gap;
c	is the concentration;
ε	is the mean deviation from the nominal width of the separating gap;
$\varkappa$	is the parameter characterizing the effect of stray convection;
$y_e = \ln q^*$;	
η	is the dynamic viscosity;
D	is the diffusivity;
$\Delta T = T_1 - T_2$	is the temperature difference between hot surface and cold surface;
$T = (T_1 + T_2)/2$	is the average temperature;
ρ	is the density;
β	is the volume expansivity;
B	is the length of separating gap.

LITERATURE CITED

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